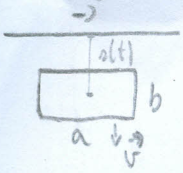
EuF 2015 - 2sem 14/03/2016

Q1.



$\Omega = R$

$s(t) = s_0 + vt$

a) $\int B\cdot dl = \mu_0 I$

$\boxed{B = \frac{\mu_0 i}{2\pi r}}$

campo entrando na região da espira

direção e sentido ?

b) $\phi = \int B\,da = \frac{\mu_0 i}{2\pi}\int_{s(t)-\frac{b}{2}}^{s(t)+\frac{b}{2}} \frac{1}{r}\cdot a\cdot dr = \frac{\mu_0 i}{2\pi}\cdot a \ln\left(\frac{\overset{\text{constante}}{\hat{s}(t)} + \frac{b}{2}}{s(t)-\frac{b}{2}}\right)$

$dA = \int_{s(t)-\frac{b}{2}}^{s(t)+\frac{b}{2}} a\,dr = a\cdot\left(\cancel{s(t)} + \frac{b}{2} - \cancel{s(t)} + \frac{b}{2}\right) = ab$

c) $\varepsilon_{in} = -\frac{d\phi}{dt} = -\frac{\mu_0 i}{2\pi} a\cdot\left(\frac{\cancel{s(t)-\frac{b}{2}}}{s(t)+\frac{b}{2}}\right)\cdot\left(\frac{v\cdot(s(t)-\frac{b}{2}) - v(s(t)+\frac{b}{2})}{(s(t)^2-\frac{b}{2})^2}\right)$

$+\frac{\mu_0 i a}{2\pi\left(s(t)^2-\left(\frac{b}{2}\right)^2\right)}\cdot vb$

$= -\frac{\mu_0 i a}{2\pi}\cdot\left[\frac{1}{s(t)+\frac{b}{2}}\cdot v - \frac{1}{s(t)-\frac{b}{2}}\cdot v\right] =$

d) $i = \frac{\varepsilon_{in}}{R}$; Regra da mão direita → $\overset{F}{\text{palma}}\ B\ v$ → sentido horário.

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Q2 $\nabla^2\vec{E} - \kappa \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{\sigma}{\varepsilon_0}\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$, sendo $\frac{1}{c^2} = \mu_0\varepsilon_0$.

a) $\vec{E}(z,t) = \vec{E}_0 e^{i(\omega t - qz)}$

$\frac{\partial E}{\partial z} = -iq E_0 e^{i(\omega t - qz)} \Rightarrow \nabla^2 E = -q^2 E_0 e^{i(\omega t - qz)}$

$\frac{\partial E}{\partial t} = i\omega E e^{i(\omega t - qz)} \Rightarrow \frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 e^{i(\omega t - qz)}$

$-q^2 \cancel{E_0 e^{i(\omega t - qz)}} + \frac{\kappa}{c^2}\omega^2 \cancel{E_0 e^{i(\omega t - qz)}} - \frac{i\sigma}{\varepsilon_0 c^2}\omega \cancel{E_0 e^{i(\omega t - qz)}} = 0$

$-q^2 + \frac{\kappa}{c^2}\omega^2 - \frac{i\sigma}{\varepsilon_0 c^2}\omega = 0$

$q = \sqrt{\underbrace{-\frac{i\sigma\omega}{\varepsilon_0 c^2}}_{i\sigma\omega\mu_0\varepsilon_0 / \varepsilon_0} + \underbrace{\frac{\kappa}{c^2}\omega^2}_{-\frac{\varepsilon}{\varepsilon_0}\mu_0\varepsilon_0}} = \boxed{\sqrt{+\varepsilon\mu_0\omega^2 + i\sigma\omega\mu_0}}$

se for isolador $\sigma = 0$

$\Rightarrow \boxed{q = \omega\sqrt{+\mu_0\varepsilon}} \Rightarrow \omega\sqrt{\mu_0\varepsilon_0\kappa} \Rightarrow \frac{\omega\sqrt{\kappa}}{c}$

b) $\tilde{q}^2 = \frac{\tilde{\kappa}\omega^2}{c^2} \Rightarrow \frac{\varepsilon\mu_0\omega^2 c^2}{\omega^2} + i\frac{\sigma\omega\mu_0 c^2}{\omega^2} = \tilde{\kappa}$

$$\boxed{\tilde{\kappa} = \underbrace{\frac{\varepsilon}{\varepsilon_0}}_{\kappa} - \frac{i\sigma}{\varepsilon_0\omega}}$$

Q2.

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$n = \frac{ck}{\omega}$

c) $\hat{\kappa} = \kappa - \frac{i\sigma}{\varepsilon_0 \omega}$ $\xRightarrow{\text{low freq}}$ $\frac{\sigma}{\varepsilon_0 \omega} \gg 1$? $\hat{n} = \sqrt{\hat{\kappa}} = \sqrt{\kappa - \frac{i\sigma}{\varepsilon_0 \omega}}$

$= i\sqrt{\frac{i\sigma}{\varepsilon_0 \omega}} =$

d) $\hat{q} = \hat{\kappa}\frac{\omega^2}{c^2}$

$\delta = \frac{c}{\omega \cdot \sqrt{\hat{\kappa}}} = \frac{c}{2\pi f \sqrt{\left(\cancel{\frac{\cdot \varepsilon}{\varepsilon_1}}^{0} - \frac{i\sigma}{\varepsilon_0 \omega}\right)}}$

low freq

Q3. 5 MeV → ← 2 MeV

a) relative velocity → how fast would you see the second body moving if in your reference frame the first body is at rest.

because photons doesn't have a rest reference frame the relative velocity is always $\underline{\underline{c}}$.

b) $E = 5 + 2 = 7$ MeV ?

c) $p = p_1 - p_2 \Rightarrow pc = \overset{5}{E_1} - \overset{2}{E_2}$ d) $E^2 = p^2 \ldots$

$$\boxed{p = \frac{3 \text{ MeV}}{c}}$$

Q4. $\sim \lambda = 10^{-10}\,m$

$\theta = 180°$

energy $\Leftrightarrow$ $E_1 + m_0c^2 = E_1' + \underbrace{\gamma m c^2}_{K}$

$E_1 - E_1' = (\gamma - 1)m_0c^2$

a) $\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) = \frac{2h}{m_0c}$.

$\Delta\lambda = \lambda' - \lambda = \frac{2h}{m_0c} \Rightarrow \boxed{\lambda' = \frac{2h}{m_0c} + 10^{-10}}$

b) $P_i = P_f$

$\frac{h}{\lambda}\hat{x} = \frac{h}{\lambda'}(-\hat{x}) + p$

$\rightarrow \hat{x}$ direção cont.

$P:$ e^-

$E' = \frac{hc}{\lambda'} \Rightarrow p' = \frac{E'}{c} \Rightarrow$ 180° $p_x' = p'$, $p_y' = 0$ $\Rightarrow$ conservação momento em x e y antes e depois

x:

$P_{i,t} = P_{f,tot} \Rightarrow \frac{h}{\lambda} = \frac{h}{\lambda'} + P_{e,x}'$

$tg\,\phi = \frac{P_{e,y}}{P_{e,x}} = 0°$ $\begin{cases} \boxed{P_{e,x}' = \frac{h}{\lambda} - \frac{h}{\lambda'}} \\ \boxed{P_{e,y}' = 0} \end{cases}$

c) $P_f = mv$

$\boxed{v = \frac{P_f}{m}}$?

$K = \frac{hc}{\lambda'} - \frac{hc}{\lambda}$

$K = \frac{1}{2}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{2K}{m}}}$

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Q5 $\gamma = \frac{c_p}{c_v}$

a)

$p = p_0 + \frac{F}{A} = \boxed{p_0 + \frac{M \cdot g}{A}}$

b) adiabatico $Q = 0$ $pV = nRT$

$du = -dW = -pdV \Rightarrow du = \left(\frac{\partial u}{\partial V}\right)_p dV + \left(\frac{\partial u}{\partial p}\right)_V dp$

$u = nc\bar{T} = \frac{\cancel{n} c pV}{\cancel{n} R}$ $\left(\frac{\partial u}{\partial V}\right)_p = -p$

$\left(\frac{\partial u}{\partial V}\right)_p = \frac{cp\,dV}{R}$

Q6.

$u(r) = k(r^2 - a^2)e^{-br^2}$

a) $[k] = \left[\frac{kg}{s}\right]$ $[y] = \left[\frac{kg\,m^2}{s}\right]$ b)

$[a] = [m]$

$[b] = \left[\frac{1}{m^2}\right]$

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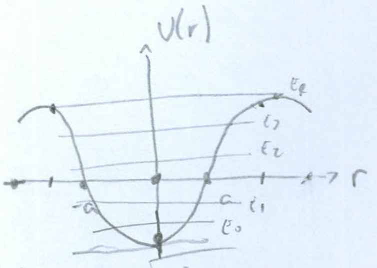
b) $\frac{dU}{dr} = \cancel{2K} a^2 b r \cancel{e^{-br^2}} + \cancel{2} r \cancel{K} \cancel{e^{-br^2}} - b \cancel{2} r^3 \cancel{K} \cancel{e^{-br^2}} = 0$

$$a^2 b r + r - b r^3 = 0 \qquad \boxed{r = 0}$$

$$r(1 + a^2 b - b r^2) = 0$$

$$\hookrightarrow r^2 = a^2 + \frac{1}{b} \Rightarrow \boxed{r = \pm\sqrt{\frac{a^2 b + 1}{b}}}$$



$$\frac{d^2 U}{dr^2} = 2K\left[a^2 b e^{-br^2} - a^2 b^2 2r^2 e^{-br^2} + e^{-br^2} - 2r^2 e^{-br^2} - b3r^2 e^{-br^2} + b r^4 2 e^{-br^2}\right]$$

$$\text{p/ } r = 0 \Rightarrow 2K\left[a^2 b + 1\right] > 0 \quad \cup$$

$$\text{p/ } r_1 = \pm\sqrt{\frac{a^2 b + 1}{b}}$$

$$\cancel{2K}\Big[a^2 b \cancel{e^{-(a^2b+1)}} - a^2 2b(a^2 b + 1)\cancel{e^{-(a^2b+1)}} + \cancel{e^{-(a^2b+1)}} - 2\left(\frac{a^2b+1}{b}\right)\cdot\cancel{e^{-(a^2b+1)}} - 3\cdot(a^2b+1)\cancel{e^{-(a^2b+1)}} + (a^2b+1)^2 2\cancel{e^{-(a^2b+1)}}\Big]$$

$$= a^2 b - a^2 2b(a^2 b + 1) + 1 - 2\left(\frac{a^2b+1}{b}\right) - 3(a^2 b + 1) + 2(a^2 b + 1)^2 = 0$$

$$= \cancel{a^2 b} - \cancel{2a^2 b} - 2a^4 b^2 + \cancel{1} - 2a^2 - \frac{2}{b} - \cancel{3a^2 b} - \cancel{3} + 2a^4 b^2 + \cancel{4a^2 b} + \cancel{2} = 0$$

$$-\cancel{2a^4 b^2} - 2a^2 - \frac{2}{b} + \cancel{2a^4 b^2} = 0 \Rightarrow < 0$$

$$U(r_1) = K\left[\left(\frac{a^2b+1}{b}\right) - a^2\right] e^{-br^2} \Rightarrow \frac{a^2 b + 1 - a^2 b}{b} \Rightarrow \boxed{\frac{1}{b} > 0}$$

$$U(r) = 0 = r^2 - a^2 \Rightarrow r = \pm a$$

c) (i) órbitas ligadas até E_4

(ii) acima dela (iii) primeiro nível (bottom)

$\boxed{6}$

d) $\vec{F} = -\vec{\nabla} V = 2rKa^2bc^{-br^2} + 2rKe^{-br^2} - 2r^3Kbe^{-br^2}$

equilíbrio ocorre em $r = 0$

$F = 0$. → harmonic oscillator

$V_1 = ar^2 + br + c$ $\qquad V_1(0) = V(0)$

$V_1' = 2ar + b$ $\qquad c = -Ka^2$

$V_1'' = 2a$ $\qquad \rightarrow b = 0$

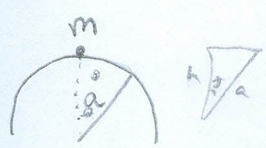
$2a = 2K[a^2b + 1]$

$a = Ka^2b + K$

$V_1 = (Ka^2b + K)r^2 - Ka^2$

$F_1 = -K_1 r = V_1' \Rightarrow K_1 = 2K[a^2b + 1]$

$w^2 = \frac{K_1}{m} \Rightarrow \boxed{f^2 = \frac{2K[a^2b+1]}{(2\pi)^2 m}}$



$\cos\theta = \frac{h}{a} \Rightarrow h = a \cdot \cos\theta$

a) $\underbrace{\mathcal{L} = \frac{1}{2}m\dot{r}^2 + mga\cos\theta}$ ou $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + mga\cos\theta$

$\frac{1}{2}ma^2\dot{\theta}^2$?? $\dot{r}$

equação de vínculo $= f = r - a = 0$

$r = a$

r:

b) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = \lambda \frac{\partial f}{\partial r}$

?

$m\ddot{r} - \lambda = 0 \quad \Rightarrow r = a \Rightarrow \boxed{\lambda = 0}$ força de vínculo

deveria ser zero quando a bola sai da esfera...

θ:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} - \lambda \cancel{\frac{\partial f}{\partial \theta}}^{\,0} = 0$

$mga\sin\theta = 0$

c) conservação?

conservado

$\frac{\partial L}{\partial r} = 0$; $\frac{\partial L}{\partial \dot{r}} = m\dot{r} = p = \text{constant}$ ... $\frac{\partial L}{\partial t} = 0 \Rightarrow$ energia conservada

d) $a(t) = a_0(1 + \cos\omega t)$

$\dot{a}(t) = -a_0 \sin(\omega t)\cdot\omega \Rightarrow L = \frac{1}{2} m a_0^2 \sin^2(\omega t)\,\omega^2 + mga\cos\theta$

$\frac{\partial L}{\partial t} \neq$, $\frac{\partial L}{\partial \dot{r}} = 0$ não é conservada ?

Q8.

a) 

$V(x,y) = \begin{cases} 0, & 0 \le x,y \le \frac{L}{2\pi} \\ \infty, & \text{out} \end{cases}$

p

$2\pi R = L$

$R = \frac{L}{2\pi}$

$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(x,y) = E\Psi$

$2\pi R = L$

$$\boxed{\frac{\hbar^2}{2m}\frac{d\Psi(x)}{dx^2} = E\,\Psi(x)}$$

$k^2 = \frac{2mE}{\hbar^2}$

b) $\frac{d^2\Psi(x)}{dx^2} + k^2\Psi(x) = 0$

$\Psi(x) = Ae^{ikx} \Rightarrow \int_0^L A^2\, dx = 1$

$$\boxed{A = \frac{1}{\sqrt{L}}}$$

$$\boxed{\begin{aligned} p &= \hbar k \\ E &= \frac{k^2\hbar^2}{2m} \end{aligned}}$$

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c) $\hat{H}\Psi(x) = E\Psi(x)$

$\Psi(x) = \Psi(x+L)$ $\cos(kL) + i\sin(kL)$

$Ae^{ikx} = Ae^{ikx} \cdot e^{ikL}$ $kL = 2\pi n$ $L = 2\pi R$

$e^{ikL} = 1 \Rightarrow e^{i2\pi n}$ $k = \frac{2\pi n}{L} \Rightarrow E = \frac{2\pi^2 n^2 \hbar^2}{L^2 m} = \boxed{\frac{n^2\hbar^2}{2mR^2}}$

$\Psi(x) = \frac{1}{\sqrt{L}} e^{i\frac{2\pi n}{L}x}$

$\Rightarrow \frac{1}{\sqrt{L}} \cos\left(\frac{2\pi n x}{L}\right)^{n=1}, n=2$

d) $E_0 = \frac{\hbar^2}{2mR^2}$ $\int_x^{\delta x + x} |\Psi(x)|^2 dx = \frac{2}{L} \cdot \cos^2\left(\frac{2\pi x}{L}\right)$

$\Psi_0(x) = \frac{1}{\sqrt{L}} e^{i2\pi\frac{x}{L}}$

$\frac{1}{L}\int_x^{x+\delta x} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{1}{L} \cdot \left[\delta x + \int_x \cos\left(\frac{4\pi x}{L}\right) dx\right]$

$= \frac{1}{L}\left[\delta x + \left(\frac{L}{4\pi} \cdot \left(-\sin\left(\frac{4\pi x}{L}\right)\right)\Big|_x^{x+\delta x}\right)\right]$?

Q9. $|\Psi\rangle = \alpha\left(|z_+^A\rangle \otimes |z_-^B\rangle - |z_-^A\rangle \otimes |z_+^B\rangle\right)$

a) $\langle\Psi|\Psi\rangle = 1$

$2\alpha^2 = 1$

$\boxed{\alpha = \frac{1}{\sqrt{2}}}$

b) Prob. direção z: $-\hbar/2^{A}$, $\frac{\hbar}{2}^{B}$ $\Rightarrow P\left(S_{1z} = -\frac{\hbar}{2}, S_{2z} = +\frac{\hbar}{2}\right) = |\langle -+|\psi\rangle|^2$

$$P(|z_-^A\rangle|z_+^B\rangle) = \boxed{\frac{1}{2}} \cdot \frac{1}{\sqrt{2}}\langle -+|\cdot(|+-\rangle - |-+\rangle) = \frac{1}{2} \checkmark$$

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c) eigenvalues of $\hat{S}_x = \pm\frac{\hbar}{2}$, measuring them means ending up in the respective eigenstate $|+\rangle_x$, $|-\rangle_x$.

Prob $\Rightarrow |{}_x\langle +|\psi\rangle|^2 \rightarrow$ os dois precisam estar na mesma base

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle) \qquad |+\rangle_y \cdot |-\rangle_y = \frac{1}{2}(|+\rangle + i|-\rangle)(|+\rangle - i|-\rangle)$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \Rightarrow \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 1\begin{bmatrix} a \\ b \end{bmatrix} \quad -ib = a \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$|\langle x|s\rangle|^2 \rightarrow$ prob of finding particle in state $|x\rangle$ given that it was in state $|s\rangle$

$$\boxed{|\pm\rangle_x = \frac{1}{\sqrt{2}}(|-\rangle \pm |+\rangle)} \Rightarrow$$

$$|{}_x\langle +|\psi\rangle|^2 = \frac{1}{\sqrt{2}}(\langle -| + \langle +|) \cdot \left(\frac{4}{5}|+\rangle + \frac{3}{5}|-\rangle\right)^{\text{sup.}} = \frac{4}{5\sqrt{2}} + \frac{3}{5\sqrt{2}} = \left(\frac{7}{5\sqrt{2}}\right)^2 = \frac{49}{50}$$

$$|{}_{x_A}\langle +, -^B|\psi\rangle = \left[\frac{1}{\sqrt{2}}(\langle +^A| + \langle -^A|)\right]\left[\frac{1}{\sqrt{2}}(\langle +^B| - \langle -^B|)\right] \cdot \left[\frac{1}{\sqrt{2}}(|z_+^A\rangle \cdot |z_-^B\rangle - |z_-^A\rangle \cdot |z_+^B\rangle)\right]$$

$$\frac{1}{8} \cdot \left[(\langle +^B| - \langle -|) \cdot (|-^B\rangle - |+^B\rangle)\right] = \frac{1}{8}[-1 - 1] = -\frac{1}{4}$$

$$\boxed{P = |\langle +^A, -^B|\psi\rangle|^2 = \frac{1}{16}} \quad ?$$

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d) $\langle z_-^A, x_+^B | \Psi \rangle = \frac{1}{2} \langle z_-^A | (\langle z_+^B | + \langle z_-^B |) \cdot (|z_+^A\rangle \cdot |z_-^B\rangle - |z_-^A\rangle \cdot |z_+^B\rangle)$

$|x_+^A\rangle = \frac{1}{\sqrt{2}} (|z_+^A\rangle + |z_-^A\rangle)$

$= \frac{1}{2} \langle z_-^A | \cdot (|z_+^A\rangle - |z_-^A\rangle) = -\frac{1}{2}$; $|\langle z_-^A, x_+^B | \Psi \rangle|^2 = \boxed{\frac{1}{4}}$

Q10.

Energias possíveis: $0, \varepsilon > 0$

a) $z = \sum_n e^{-\beta \varepsilon n} = 1 + e^{-\frac{\varepsilon}{k_B T}}$; $Z = z^N = (1 + e^{-\frac{\varepsilon}{k_B T}})^N$

$\ln Z = N \ln (1 + e^{-\frac{\varepsilon}{k_B T}})$ $\sim e^{-\beta \varepsilon}$

$F = -k_B T \cdot N \ln (1 + e^{-\frac{\varepsilon}{k_B T}}) \Rightarrow E = -\frac{N \cdot e^{-\beta \varepsilon}}{1 + e^{-\beta \varepsilon}} (-\varepsilon) ; \frac{E}{N} = \frac{\varepsilon e^{-\frac{\varepsilon}{k_B T}}}{1 + e^{\frac{\varepsilon}{k_B T}}}$

$\frac{S}{N} = -\left(\frac{\partial F}{\partial T}\right)_V = k_B \ln (1 + e^{-\frac{\varepsilon}{k_B T}}) + \frac{k_B T \cdot (e^{-\frac{\varepsilon}{k_B T}} \cdot (\frac{\varepsilon}{k_B T^2}))}{(1 + e^{-\frac{\varepsilon}{k_B T}})}$

$\boxed{= k_B \ln (1 + e^{-\frac{\varepsilon}{k_B T}}) + \frac{1}{T} \left(\frac{E}{N}\right)}$ pode estar errado

b) $$\frac{E}{N} = \frac{\varepsilon e^{-\frac{\varepsilon}{k_B T}}}{1 + e^{-\frac{\varepsilon}{k_B T}}}$$

c) $$\frac{E}{N} = \frac{\varepsilon}{e^{\frac{\varepsilon}{k_B T}} + 1} \Rightarrow \frac{dE}{dT} = - \frac{\varepsilon \left[e^{\frac{\varepsilon}{k_B T}} \cdot \left(- \frac{\varepsilon}{k_B T^2} \right) \right]}{\cancel{\left(e^{\frac{\varepsilon}{k_B T}} + 1 \right)^2}} = 0$$

$$\frac{e^{\frac{\varepsilon}{k_B T}} \cdot \varepsilon^2}{k_B T^2} = 0 \Rightarrow T \to \infty$$

?